

Fluctuation theorems for quantum maps

Gonzalo Manzano^{1,2}, Jordan M. Horowitz³, and Juan M. R. Parrondo¹

¹ Departamento de Física Atómica, Molecular y Nuclear and GISC,
Universidad Complutense Madrid, 28040 Madrid, Spain

² Instituto de Física Interdisciplinar y Sistemas Complejos IFISC (CSIC-UIB),
UIB Campus, E-07122 Palma de Mallorca, Spain

³ Department of Physics, Massachusetts Institute of Technology,
400 Technology Square, Cambridge, Massachusetts 02139, USA

When considering small systems, quantum fluctuations, in addition to thermal ones, come into play. Furthermore, there is a wide range of phenomena without classical counterpart that cannot be neglected, such as coherence, squeezing or entanglement, in both single and many-body systems. A promising route to the understanding of thermodynamics in quantum systems are the fluctuation theorems, which establish exact statements about the fluctuations of thermodynamic quantities such as work, heat or entropy production, in systems arbitrarily far from equilibrium [1].

Work fluctuation theorems have been extensively investigated in the quantum regime under an inclusive Hamiltonian approach. Also fluctuation theorems for the exchange of heat and particles in transient and steady-state regimes has have been established within the so-called two-measurements-protocol [2, 3]. This framework has provided important results, but its rigid assumptions restrict possible extensions to more general situations, apart from being impractical in most physical situations. On the other hand, an alternative approach recently considered is the derivation of fluctuation theorems for arbitrary completely-positive and trace-preserving (CTPT) maps, as they provide a compact description of general physical processes condensing the main effects of the environmental action in a set of few relevant variables.

Here we present a novel fluctuation theorem valid for a broad class of quantum CPTP maps [4]. It is based in the concept of a *nonequilibrium potential*, an intrinsic fluctuating property of the map which allows the thermodynamic description at the single trajectory level in most situations of interest. Our theorem goes beyond previous results for specific classes such as *unital* maps —maps preserving the identity operator— [5, 6], or theorems limited by efficacy (correction) terms [7, 8].

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