Zealots in the mean field noisy voter model

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The dynamical evolution of opinions is frequently studied using the voter model [1, 2, 3], which usually considers a set of voters or agents supporting two opinions that change due to the influence to all of them (mean–field level), in such a way that a voter supporting an opinion changes it with a rate proportional to the fraction of agents holding the opposite opinion. The model exhibits a competition that ends up after a transient time to consensus (if the system is finite), the two possible absorbing states being equiprobable. This macroscopic picture may change when the model is modified in order to account for more realistic situations, such as the heterogeneity and the free will of agents.

The heterogeneity in the population of agents or units is usually reflected in two different aspects, namely, in the intrinsic properties of the agents and in the structure of the interactions. In the first case, agents may be differentiated by its intrinsic rates of change between states, an extreme case corresponding to a system of equal agents but one that does not change opinion, i.e. a zealot. In the second case, some agents are influenced only by some others in the processes of opinion changing, the system being embedded in a graph or network of interactions. Many studies incorporate both ingredients to the voter model at the same time. First studies focused on the influence of few zealots in the case of regular networks, as well as in the case of all-to-all interactions, or the so–called mean–field approximation [4, 5, 6]. In these cases, the presence of zealots changes drastically the evolution of the system. If only one zealot is present, the system approach much faster one consensus state, the one that corresponds to the zealot. When the number of zealots of different opinions are the same, the system reaches a steady state where the two opinions coexist. More recently, the effect of complex network has also been analyzed, with similar conclusions [7, 8].

The voter model has also been modified to account for the free will of the voters, leading to the so–called noisy voter model [9] or Kirman model [10]. Now, the rate at which one agent changes opinion not only depends on the fraction of opposite voters, but on an intrinsic constant, the free–will parameter. The main difference of the present model with respect to the original one is that the free will avoids the system from reaching the consensus states. Moreover, the system undergoes a finite-size transition, by increasing the free–will constant, form a bimodal behaviour, where agents spend most of the time close to the consensus states, to a unimodal one, where a non-negligible fraction of agents are at different states [11, 12]. Once again, the presence of a complex networks seems to respect the latter picture, while the critical values are modified. Few studies cover mixture of different agents in the context of the noisy voter model, see [13] as an exception, nor the influence of zealots.

In this work, we analyze the influence of zealots on the noisy voter model, focusing on the steady–state properties. We also provide a deep relation between this problem and a system made of heterogeneous voters. More precisely, we consider three cases of interest: (a) one optimistic zealot influencing a sub–population, (b) two opposite zealots influencing a sub–population, and (c) two opposite zealots influencing different sub–populations of the system. The main results are summarized in a phase diagram $(a/h, N_1/N)$ where a/h is the ratio between free–will and herding coefficients and N_1/N is the fraction of affected agents. The noise voter model hast two phases separated by an horizontal line corresponding to $a/h = 1/N$. The latter line splits into two ones in case (a) while it turns curved in cases (b) and (c).

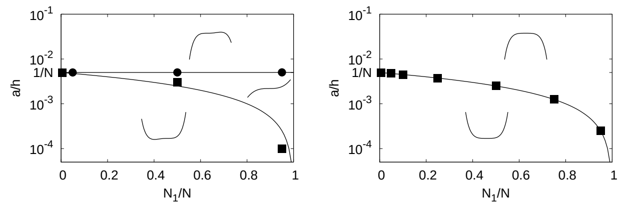

Figure 1: Phase diagram for cases (a) (left) and (b) and (c) (right).

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