

# Model of nonlocal birth-death competition with volume exclusion

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We propose a stochastic birth-death competition model for particles with excluded volume. The particles move, reproduce, and die on a regular lattice. While the death rate is constant, the birth rate is spatially nonlocal and implements inter-particle competition by a dependence on the number of particles within a finite distance. The finite volume of particles is accounted for by fixing an upper value to the number of particles that can occupy a lattice node, compromising births and movements. In this way, we extend a previous model [1, 2] of competition between individuals of negligible size. Examples of the spatiotemporal configurations obtained with the model proposed here [3] in one spatial dimension can be seen in Figure 1.

We derive closed macroscopic equations for the density of particles and spatial correlation at two adjacent sites. Under different conditions, the description is further reduced to a single equation for the particle density that contains three terms, namely diffusion, a linear death, and a highly nonlinear and nonlocal birth term.

Steady-state homogeneous solutions, their stability which reveals spatial pattern formation, and the dynamics of time-dependent homogeneous solutions are discussed and compared, in the one-dimensional case, with numerical simulations of the particle system.

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- [1] E. Hernández-García, C. López, *Clustering, advection and patterns in a model of population dynamics*. *Physical Review E* **70**, 016216 (2004).
  - [2] E. Hernández-García, E. Heinsalu, C. López, *Spatial patterns of competing random walkers*, *Ecological Complexity* **21**, 166-176 (2015).
  - [3] N. Khalil, C. López, E. Hernández-García, *Nonlocal birth-death competitive dynamics with volume exclusion*, *ArXiv:1701.06196* (2017).

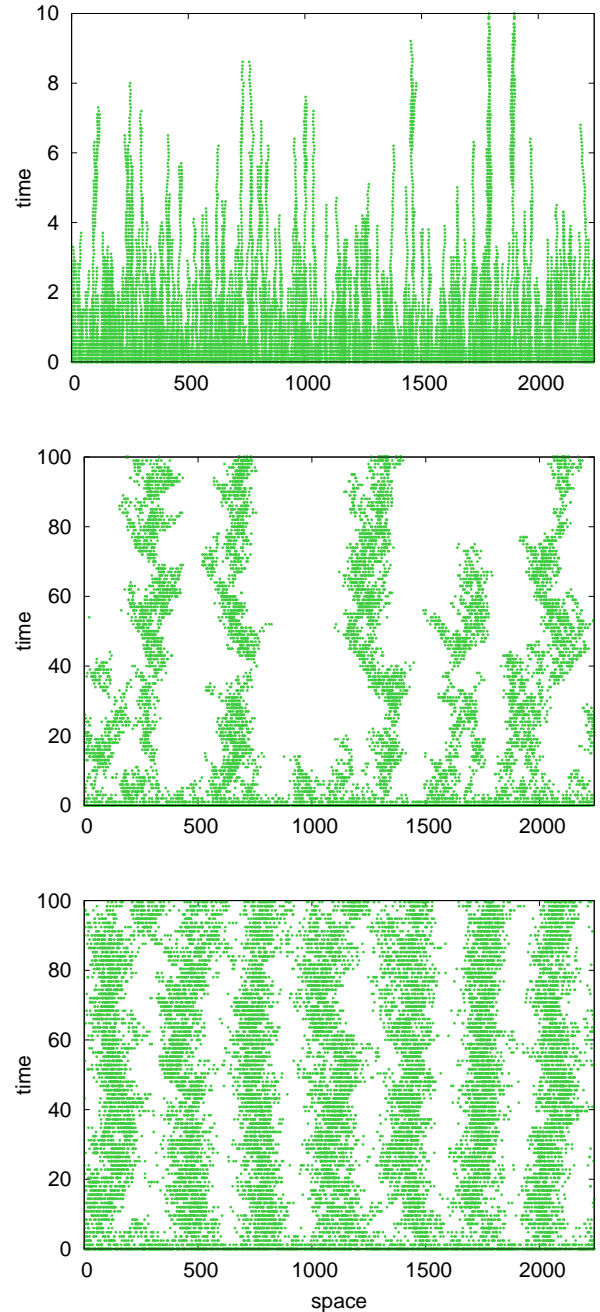


Figure 1: Three spatio-temporal configurations of the one-dimensional interacting particle system under different parameters. Green dots are the particle positions. The first panel shows extinction, the last one the emergence of a spatially periodic density state. The middle panel shows an intermediate noisy behavior.