# Hierarchical invasion of cooperation in complex networks

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The emergence and survival of cooperation is one of the hardest problems still open in science [1]. Several factors such as the existence of punishment, fluctuations in finite systems, repeated interactions and the formation of prestige may all contribute to explain the counter-intuitive prevalence of cooperation in natural and social systems. The characteristics of the interaction networks have been also signaled as an element favoring the persistence of cooperators [2]. Here we consider the invasion dynamics of cooperative behaviors in complex topologies (in particular, scale-free and random networks). The invasion of a heterogeneous network fully occupied by defectors is performed starting from nodes with a given number of connections (degree) $k_0$. The system is then evolved within a Prisoner's Dilemma Game (PDG), through Unconditional Imitation (UI) or Replicator (REP) evolution rules, and the outcome is analyzed as a function of $k_0$ and the degree $k$ of the nodes adopting cooperation. The payoff matrix of the PDG is

$$\hat{P} = \begin{array}{|c|c|c|} \hline & \textbf{C} & \textbf{D} \\ \hline \textbf{C} & 1 & 0 \\ \hline \textbf{D} & 1.4 & \varepsilon \\ \hline \end{array},$$

where $\varepsilon$ is the punishment.

Carried out using both numerical and analytical approach, our results show that the invasion proceeds following preferentially a hierarchical order in the nodes from those with higher degree to those with lower degree, as shown in Fig. 1 in the case of scale-free topology (similar results hold on random networks).

However, the invasion of cooperation will succeed only when the initial cooperators are numerous enough to form a cluster from which cooperation can spread. This implies that the initial condition must be a suitable equilibrium between high degree and high numerosity, which usually takes place, when possible, at intermediate values of $k_0$ (see Fig. 2). These findings have many potential real-world applications, as they suggest that, in order to promote cooperative behavior on complex networks, one should infect with cooperators high-but-not-too-high degree nodes.

More details are available in Ref. [3].

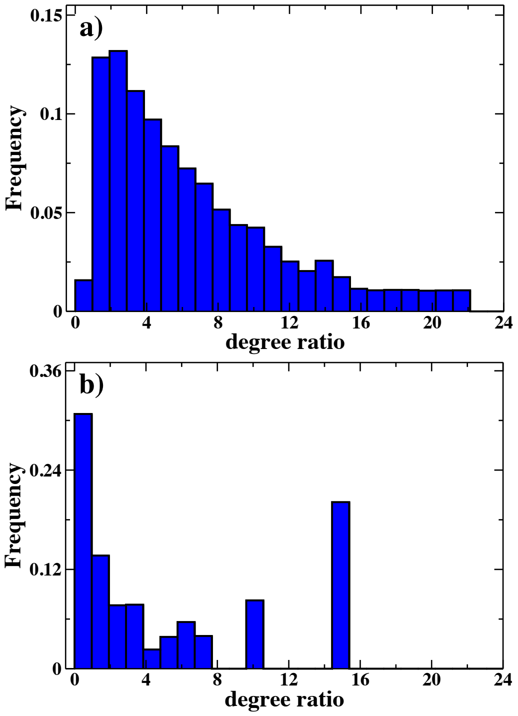

Figure 1: Histograms of the frequency of transitions from defection to cooperation as a function of the ratio $k_i/k_f$, being $k_f$ the degree of the agent which flipped from defection to cooperation by imitating the agent with degree $k_i$, for a system on a scale free network (exponent $\beta = 1.6$ and size $N = 2000$, $\varepsilon = 0.05$ and $k_0 = 30$), in case of a) UI evolution rule, and b) REP updating. The cumulative frequency of the transitions with degree ratio larger than one (i.e. the top-down invasion acts) is $\simeq 98\%$ for UI and almost $70\%$ for REP. From Ref. [3].

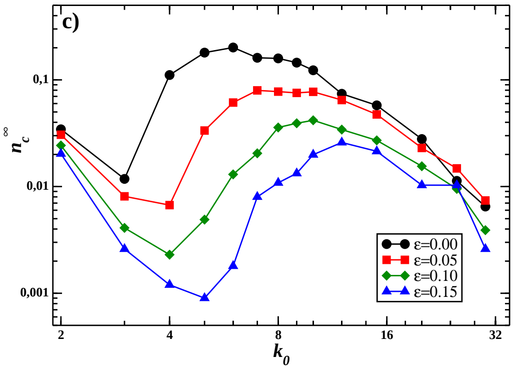

Figure 2: Final cooperator density as a function of the invasion degree $k_0$ for a system on a scale free network (size $N = 1000$, exponent $\beta = 1.6$).

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[1] E. Fehr and U. Fishbacher, Nature **425**, 785 (2003).

[2] A. Nowak and R. M. May, Nature **369**, 826 (1992).

[3] D. Vilone, V. Capraro, and J. J. Ramasco, preprint, arXiv:1701.03710 [physics.soc-ph] (2017).