Reconstructing networks of pulse-coupled oscillators from non-invasive observations

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We present a method for reconstructing a network of pulse-coupled oscillators from non-invasive observations of the system's output. Assuming that the pulse trains of all nodes are known and that the coupling between the elements is sufficiently weak to justify the phase dynamics description, we recover the connectivity of the network and properties of the nodes.

In our previous work, we have developed an approach to recover the connectivity of a network in the case of oscillatory processes that allow for conventional phase estimation [1]. In this work, we have tackled the problem for the case of pulse-coupled oscillators. Our basic model for the network nodes are phase oscillators which issue a spike when their phase φ reaches 2π. (We consider the phases in the $[0, 2\pi)$ interval, i.e., after the spike generation the phase of the unit is reset to zero). This spike affects all other units of the network according to the strength of the corresponding connections. Let the size of the network be N and let the connectivity be described by an $N \times N$ coupling matrix $\mathcal{E}$, whose elements ϵ_{km} quantify the strength of the coupling from unit m to unit k. Between the spiking events, phases of all units obey $\dot{\varphi}_k = \omega_k$, where ω_k are natural frequencies. If unit k receives a spike from oscillator m, then it reacts to the stimulus according to its phase response curve (PRC) [2], $Z_k(\varphi)$. This means that the phase of the stimulated unit is instantaneously reset, $\varphi_k \to \varphi_k + \epsilon_{km} Z_k(\varphi_k)$.

Our approach is based on making a preliminary estimation of the network connectivity by evaluating the impact an oscillator has on another oscillator's inter-spike intervals. This estimation, although crude, gives us some insight into the network, such that together with linearly approximating the phases and representing the PRC as a finite size Fourier series, we can get an approximation for the PRC. Once both approximations (connectivity and PRC) are obtained, they can be used to better approximate phases, which in turn yields better approximations of connectivity and PRC in the next iteration of the process. The more iterations one does, the better the recovery (see Fig. 1 for an example of the reconstruction).

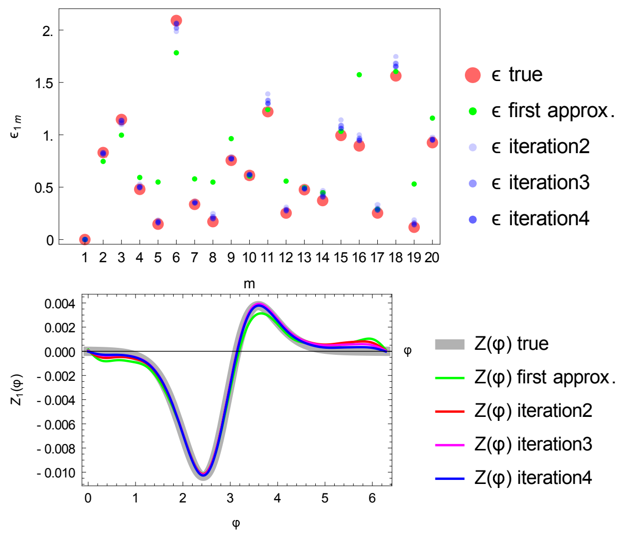

Figure 1: The reconstruction of the connectivity ϵ (top) and phase response curve $Z(\varphi)$ (bottom) of the first node in a network of 20 oscillators. Notice how the reconstruction improves with subsequent iterations (in different colors).

[1] B. Kralemann, A. Pikovsky, and M. Rosenblum, *Reconstructing effective phase connectivity of oscillator networks from observations*, New Journal of Physics **16**, 085013, 2014.

[2] D. Hansel, G. Mato, C. Meunier, *Synchrony in Excitatory Neural Networks*, Neural Comput. **7(2)**, 307-337, 1995.