Immunization and targeted destruction of networks using explosive percolation

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Immunization of networks against epidemic spreading is an important topic in complex-systems. Some applications of this area include prevention of infectious diseases, computer safety strategies against malicious viruses and information spreading in social networks. Infectious spreading in a population use the network of contacts between nodes for their spread. Accordingly, immunization corresponds to an attack that fragments the network on which it can spread. We propose a new method, 'explosive immunization' (EI) [1], to find those nodes whose removal is most efficient in destroying connectivity. Vaccinating such nodes provides an efficient way to fragment the network and reduce the possibility of large epidemic outbreaks. While most of the works assume that important blockers are equivalent to good spreaders [2], our strategy specifically targets the first kind, improving thus the results compared to other immunization methods.

In our approach, the network consists of N nodes, out of which qN are vaccinated; the rest are left susceptible to the infection. The size of an invasion will depend on the fraction q of immunized nodes, the type of epidemic and its virulence. However, the maximum fraction of nodes infected at any time will always be bounded by the relative size $S(q)$ of the largest *cluster of susceptible* nodes. Keeping $S(q)$ as small as possible will therefore ensure that epidemic outbreaks of any type are as small as they can be for a vaccination level q. For large networks the aim of immunization is to fragment them so that $S(q) = 0$. The immunization threshold q_c is the smallest q-value at which $S(q) = 0$. Although q_c is not well defined for finite networks, it can be estimated reliably. Below this value, the existence of a giant cluster is unavoidable, thus the problem is reformulated in identifying the nodes that minimize $S(q)$.

Contrary to most of the works, we start from an inverse approach, where all the nodes are vaccinated so there is no risk of an epidemic. At each step we "unvaccinate" the node with less "blocking ability" among a finite set of randomly chosen candidates. This is directly related to the concept of *explosive percolation* proposed by Achlioptas *et al.* [3]. Explosive percolation has been discussed in a large number of papers because of its very unusual threshold behavior but, to our knowledge, our work is the first problem where it is practically used. We grade the blocking ability of a node using two different heuristic scores, depending on which side of the critical threshold q_c we are. The first score (for $q > q_c$) uses the size of the cluster that each node would join if we add it to the network, together with a parameter proportional to the node degree. When the giant cluster emerges, this method grows secondary large clusters that eventually will join the largest cluster. The cluster merging generates then an undesired explosive behavior (see red dashed line in figure 1). In order to prevent this situation, below q_c we use a second score that specifically forces the growth of the largest cluster, but in a minimal way. This avoids the formation of secondary large clusters and minimizes $S(q)$ for $q < q_c$ (see continuous black line in figure 1).

We tested Explosive Immunization in several random models and real world networks. Our results are compared with the outcome of the Collective Influence method (CI) [2] among others. Although CI had been claimed to be the best immunization method in the literature, EI is better in detecting the critical threshold q_c (see figure 1). In general it also provides the minimal $S(q)$ below the threshold: although it is not optimal everywhere in real-world networks, a combination of the two scores always provides the bests global results. In addition, our method is also extremely fast with time complexity linear in N up to logarithms.

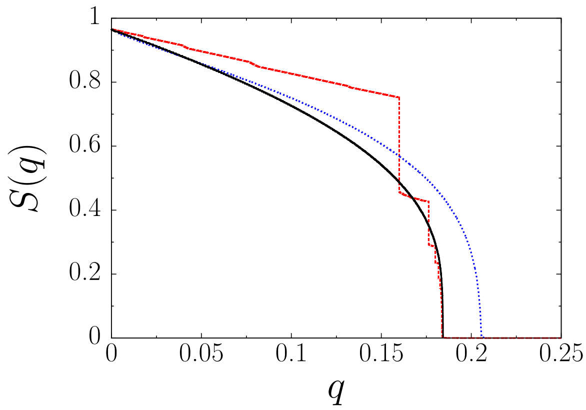

Figure 1: Relative size $S(q)$ of the largest clusters against q, for a Erdös-Rényi network with $N = 10^6$ and average degree 3.5. The red dashed curve with jumps is obtained if EI is used with the first score for all q. The continuous black curve is obtained using the second score for $q < q_c$. The blue dotted line shows the results using CI from [2]. EI estimates $q_c \simeq 0.1838$.

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