Chaotic synchronization in circular lattices

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We investigate the relationship between the emergence of chaos synchronization in coupled dynamical systems and the properties of the spatial network where they are embedded. By employing a general model of N coupled chaotic maps in a ring lattice with $2k$ edges (each map is connected to its k nearest neighbors on either side) for such systems, we found the minimal conditions for achieving synchronization. Since we are particularly interested in chaos synchronization, we choose the logarithmic map $f(x_t^i) = b + \ln|x_t^i|$, where b is a real parameter and $x_t \in (-\infty, \infty)$. This map exhibits robust chaos with no periodic windows and no separated chaotic bands, on the interval $b \in [-1, 1]$. The synchronization in these extended systems is characterized by the asymptotic time-average $\langle\sigma\rangle$ (after discarding a number of transients) of the instantaneous standard deviations σ_t of the distribution of state variables x_t. Varying the number of elements N in the network, the edges for each node and the value of the coupling parameter ε, we found the regions where the synchronized states emerge on the space of parameters of the system. The linear stability analysis, employing the eigenvalues determined of the associated coupling matrix to the system, allows to get the stability condition for the synchronized states. Fig. 1 shows the region for synchronized states (blue), given by $\langle\sigma\rangle = 0$ and by the stability condition (green and red lines), on the space of parameters $(2k, \varepsilon)$ for a ring of $N = 500$.

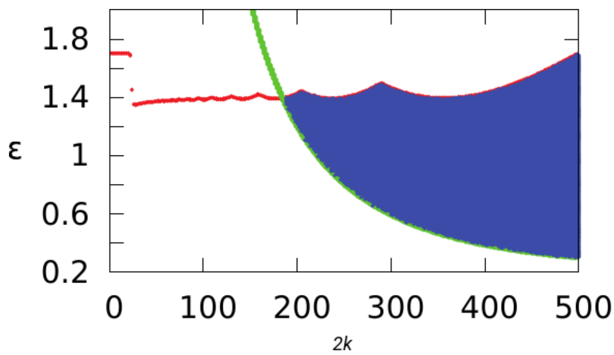

Figure 1: Boundaries $\langle\sigma\rangle = 0$ on the plane $(2k, \varepsilon)$ for synchronization (blue region). The green and red lines are obtained by the stability condition.