Role of centrality measures in a dynamic model of competences adquisition in time-dependent networks

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How humans learn has been an open question over centuries. And still it is. The secrets of competence and knowledge acquisition remain hidden despite the efforts of experts in many fields. Qualitative studies and noisy data is not enough to predict and improve the setting which optimize ability growth. We suggest a model of performance evolution which may help us describe, quantify and better analyse this open question. We cannot neglect the apparent connection between different factors within one single individual. Our starting point is a model [1] which considers the effect of variables interacting between each other:

$$\frac{\Delta x_i}{\Delta t} = \left(r_i x_i \left(1 - \frac{x_i}{K_i}\right) + \sum_{n.n} s_j x_i x_j \right) \left(1 - \frac{x_i}{C_i}\right) \quad (1)$$

Moreover, we want to stress the effect of the interactions between different individuals, which we also ought to consider. Therefore, a graph of graphs is naturally the best representation of this type of systems, considering these two type of networks, both characterized by its topology and temporal scale and evolution.

$$\begin{aligned}\frac{\Delta x_{ij}}{\Delta t} = \Bigg(& r_{ij} x_{ij} \left(1 - \frac{x_{ij}}{K_{ij}}\right) + \sum_{k \neq j} s_{kj} x_{ij} x_{ik} \\ & + \sum_{l \neq i} w_{li} x_{ij} x_{lj} \Bigg) \left(1 - \frac{x_{ij}}{C_i}\right)\end{aligned} \quad (2)$$

Equation (2), besides a logistic self-growing term, considers the effect of the network of individuals and the effect of the inner network of variables for each single individual. Furthermore, we have introduced two adjacencies matrices concerning the intensities of individuals influence and the intensity of variables influence. To start with a simpler system we focus on the system described by Equation (1). We aim to prove the connection between the topology of a network and its dynamics, described by a system of coupled non-linear differential equations. The main contribution is establishing a direct correlation of certain centrality measures and the optimal performance. Katz centrality or alpha centrality can be rewritten as the solution of one of the stable solutions of the dynamic system as long as certain conditions hold.

The stability of the system depends on both the topology and the dynamics. But, how general is this result? We want to study whether the effect of time in time-dependent networks allow us to stablish a similar connection between performance and centrality measures as well as allowing more freedom on parameters.

The relation of centrality measures and performance may lead us to predict the best achievements on the long run by just seeking the best configuration according to the centrality. However, when we relax some of the constrains the correlation with Katz centrality is no longer valid. We thus introduce a new centrality measure which takes into account not only topology but dynamics. This new centrality correlates with the new stable solutions.

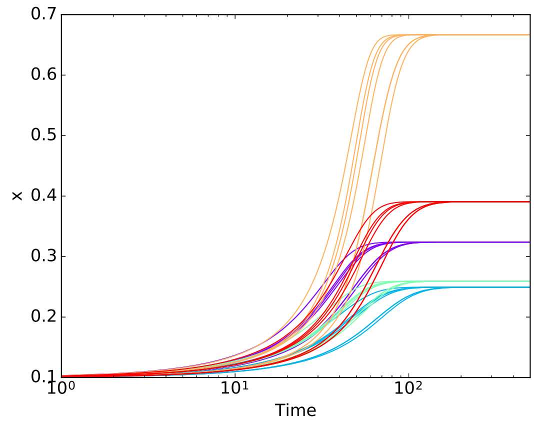

Figure 1: Temporal evolution of performance for 7 individuals and 5 inner variables

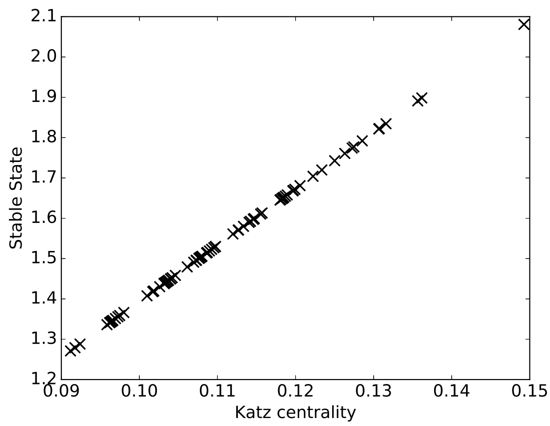

Figure 2: Stable state values as a function of the Katz centrality of the nodes, considering a random network of 80 nodes linked by 180 edges

[1] Ruud J. R. Den Hartigh et al. *A Dynamic Network Model to Explain the Development of Excellent Human Performance,* (2016).

[2] Deanna Blansky et al. *Spread of Academic Success in High School Social Network* (2015).

[3] Julliette Stehl et al. *High-Resolution Measurements of Face-to-Face Contact Patterns in a Primary School* (2011).