We introduce a coarse-grained stochastic model for the spontaneous activity of neuronal cultures to explain the phenomenon of noise focusing [1], which entails localization of the noise activity in excitable networks with metric correlations. The system is modeled as a continuum excitable medium with a state-dependent spatial coupling that accounts for the dynamics of synaptic connections. The most salient feature is the emergence at the mesoscale of a vector quantity $V(r)$, the avalanche field, that acts as an advective carrier of the activity noise. This entails an explicit symmetry-breaking of isotropy and homogeneity that stems from the amplification of the quenched fluctuations of the network by the activity avalanches, concomitant with the excitable dynamics. We discuss the microscopic interpretation of $V(r)$, and propose an explicit construction of it. The coarse-grained model shows excellent agreement with simulations at the network level and with experimental data from neuronal cultures.

The purpose of this work is to transcend the qualitative picture emerging from [1] where noise focusing was first reported, by developing a theoretical framework that enables a deeper quantitative understanding of that phenomenon, and its contextualization from the perspectives of excitable systems, constructive effects of noise and transport in complex networks. The proposed model reproduces the experimental observations in cultures and at the same time provides insights on the origin of the inherent symmetry-breaking emerging from the network quenched disorder.

As in Anderson localization, the phenomenon here elucidated results from a modification of transport properties due to disorder, even though the mechanisms are completely different. As opposed to the wave interference of multiple scattering competing with diffusive transport, in our case it is the emergence of ballistic transport that competes with diffusion. Nonetheless, in both cases the localization is a collective (nonlocal) effect of disorder in extended regions, as opposed to eventual trapping in locations with special properties of the disorder.

The phenomenon of noise focusing is generic in relatively simple excitable networks, such as in neuronal cultures, but its relevance to in vivo neuronal tissues remains to be established. From a fundamental point of view, the direct quantitative prediction of the symmetry-breaking avalanche field for a specific network, given a set of dynamical equations, remains a nontrivial open question.