Synchronization of fluctuating delay-coupled chaotic networks

Otti D'Huys[1], Manuel Jiménez-Martín[2], Javier Rodríguez-Laguna[2] and Elka Korutcheva[2,3]

[1]Department of Mathematics, Aston University, B7 4ET Birmingham, United Kingdom
[2]Departamento de Física Fundamental, UNED, Spain
[3]G. Nadjakov Inst. Solid State Physics, Bulgarian Academy of Sciences, 1784, Sofia, Bulgaria

Networks with coupling delays play an important role in various systems, such as coupled semiconductor lasers, traffic dynamics, communication networks, genetic transcription circuits or the brain [1]. Such a coupling delay typically arises due to the finite velocity of transmission of information. Here, we study the effect of a connectivity that changes over time. Such network fluctuations are essential features of some real networks, as for instance, interacting neurons, where synaptic plasticity continuously changes the topology [2].

We concentrate on the synchronization properties of chaotic maps, coupled with an interaction delay T_d. The coupling topology fluctuates between an ensemble of directed small-world networks, while keeping the mean degree constant. These network fluctuations are random, and not adaptive, i.e., the network evolution is not linked to the state in any way.

We find that the synchronization properties of the fluctuating network are strongly dependent on the two time scales T_d, the delay along the links, and T_n, the timescale of the network fluctuations. When the network fluctuations are much faster than the coupling delay ($T_n \ll T_d$) the synchronized state can be stabilized by the fluctuations: synchronization can be stable even if most or all temporary network topologies are unstable. There is a qualitative agreement with the fast switching approximation [3], however not a quantitative agreement. We complement these results with analytical findings on small fluctuating motifs.

When we increase T_n, the synchronized state destabilizes as both time scales collide ($T_n \approx T_d$). Synchronization is more probable as the network time scale increases further. However, in the slow network regime ($T_n \gg T_d$) we find that the long term dynamics is desynchronized whenever the probability of reaching a non-synchronizing network is finite. Indeed, if the network acquires a desynchronizing configuration, it evolves sufficiently far away from the synchronized state, and the probability that subsequently sampled synchronizing networks take the system back to synchronization is negligible.

Our results are demonstrated in Fig. 1, which shows the average the synchronization level $\langle \mathcal{S} \rangle$ (defined as the negative logarithm of the standard deviation over the nodes) in networks of Bernoulli and logistic maps, for an average of 1000 realizations. For both types of maps, and both for ensembles that are on average synchronizing and non-synchronizing, we find similar behavior. For short fluctuation times ($T_n = 1$ or 10), the average synchronization level remains close to its maximum value. As the fluctuation time increases ($T_n = 50$, $T_n = 100$), the synchronization level decreases. It increases again as the fluctuation time increases further ($T_n = 500$ and $T_n = 1000$). Eventually, in the slow network regime ($T_n = 5000$, $T_n = 10000$) the system desynchronizes fast. The oscillatory behavior results from our choice of initial perturbation.

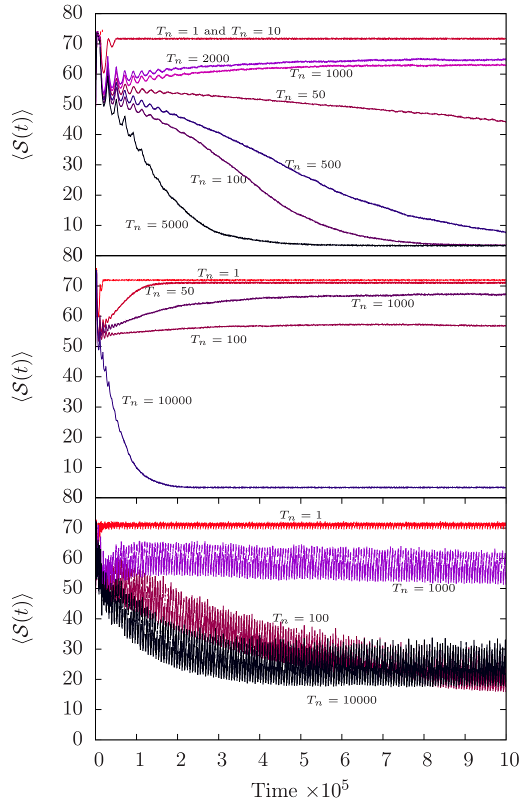

Figure 1: Realization average of the synchronization level, $\langle \mathcal{S}(t) \rangle$, over $N_s = 1000$ samples, as a function of time for different systems. Top: Bernoulli system, $N = 40$, the mean degree is $k = 1.5$ (in average, non-synchronizing), for different values of T_n. Center: Bernoulli system with $N = 40$ sites, mean degree is $k = 1.8$ (in average, synchronizing), for different values of T_n. Bottom: logistic system, $N = 40$ and mean degree $k = 1.5$ (in average, non-synchronizing). In all cases, $T_d = 100$, and we only show time steps which are multiples of 100.

[1] T. Erneux, textitApplied Delay Differential Equations, Springer New York (2009).

[2] D. V.Buonomano and M. M. Merzenich, Annual Review of Neuroscience **21**, 149 (1998).

[3] D. J.Stilwell, E. M. Bollt and D. G.Roberson, SIAM Journal on Applied Dynamical Systems **5**, 140 (2006).