Statistical signatures of photon-subtracted highly multimode states

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Quantum entanglement, one of the key resources for quantum information processing, can be deterministically generated in a scalable manner in continuous variable (CV) optical systems [1]. However such CV entangled states typically display Gaussian statistics, which limits their use for quantum computing [2]. It is experimentally feasible to overcome this problem by subtracting photons from multimode Gaussian states, and thus make them non-Gaussian [3]. In the present contribution we use techniques from quantum statistical mechanics [4] to develop a new characterisation scheme for such *multimode* photon-subtracted states of light. Our aim is to characterise non-Gaussian properties of these states by ideally measuring only a small number of observables.

In the CV regime it is natural to associate a $2m$-dimensional phase space to an m-mode system. The observables acting on this phase space are the field quadratures, $Q(f)$, which fulfil the general commutation relation $[Q(f), Q(g)] = 2i(f, Jg)$. Here f and g are arbitrary vectors in the optical phase space, and J is a matrix which represents the symplectic structure.

The states which we consider are non-displaced Gaussian states, ρ_G, from which a single photon is subtracted in a mode-selective way [3]. Such a photon-subtracted state can be described by

$$\rho = \frac{1}{\mathrm{tr}\{a^\dagger(g)a(g)\rho_G\}} a(g)\rho_G a^\dagger(g), \tag{1}$$

where $a(g)$ is the annihilation operator for a specific mode g. The normalisation factor $\mathrm{tr}\{a^\dagger(g)a(g)\rho_G\}$ is directly related to the success probability for heralding a photon.

To analyse these states, we study *truncated correlation functions* [4], $\langle Q(f_1)\dots Q(f_\nu)\rangle_T$. These quantities are particularly useful in the present context because for *any* Gaussian state, we find that

$$\langle Q(f_1)Q(f_2)\dots Q(f_\nu)\rangle_T = 0, \text{ for } \nu > 2. \tag{2}$$

In other words, for any non-Gaussian state we can find non-zero truncated correlations for orders higher than two. The centrepiece of this work is to analyse the behaviour of these truncated correlations for the photon-subtracted states as given in (1), and to investigate schemes by which they can be extracted from experiments.

The initial Gaussian state ρ_G is fully characterised by its *covariance matrix* V. However, the subtraction of a photon induces additional correlations, which is already apparent in the second-order truncated correlation functions

$$\langle Q(f_1)Q(f_2)\rangle_T = (f_1, Vf_2) + i(f_1, Jf_2) + A_g(f_1, f_2). \tag{3}$$

The function A_g, which describes the induced correlations between pairs of modes, depends strongly on the mode g from which the photon was subtracted.

Our key result is that this function A_g also describes all higher order truncated correlation functions. For example, one obtains that

$$\begin{aligned}\langle Q(f_1)Q(f_2)Q(f_3)Q(f_4)\rangle_T & \\ = &- A_g(f_1, f_2)A_g(f_3, f_4) \\ &- A_g(f_1, f_3)A_g(f_2, f_4) \\ &- A_g(f_1, f_4)A_g(f_2, f_3),\end{aligned} \tag{4}$$

which is in general non-zero.

We will present the *general analytical expression* for the function A_g, which can be applied to any photon-subtracted Gaussian states for which condition $\langle Q(f)\rangle_T = 0$ holds. Moreover, we elaborate on the role of these truncated correlation functions as an *experimentally feasible* tool to certify that the state cannot be represented by any mixture of Gaussian states. Finally, we analyse properties of the state's Wigner function through our understanding of the truncated correlation functions.

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