Dynamical localization in realistic kicked rotors

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We will show at the conference that dynamical localization (quantum suppression of classical diffusion) in the context of ultracold atoms in periodically shaken optical lattices subjected to time-periodic modulations having equidistant zeros depends on the impulse transmitted by the external modulation over half-period rather than on the modulation amplitude. This result provides a useful principle for optimally controlling dynamical localization in general periodic systems, which is capable of experimental realization.

Quantum effects in transport phenomena in classical systems represent an interesting fundamental issue in quantum theory that started from a remark of Einstein in his celebrated paper on torus quantization. One of such effects, widely studied in the context of time-periodic systems, is the quantum suppression of classical chaotic diffusion, or dynamical localization (DL) for short. Remarkably, this effect is a quantum manifestation of the fact that a time-periodic modulation can stabilize a system, and it is thus expected to play a key role in our understanding of the problem of quantum-classical correspondence in classically chaotic systems. While it is natural to think that, with the period fixed, this effect must depend on the temporal rate at which energy is transferred from the driving mechanism to the system, i.e., on the modulation waveform, the main target of study up to now has only been their dependence on the modulation amplitude because of the traditional use of sinusoidal modulation. Recent work has provided strong evidence for a different dependence of DL on sinusoidal and square-wave modulations. Since there are infinitely many different waveforms, a natural question arises: How can the influence of the shape of a periodic modulation on DL be quantitatively characterized. In this work, we demonstrate that for space-periodic systems subjected to a generic AC time-periodic modulation with equidistant zeros such characterization is well provided by a single quantity: the impulse transmitted by the modulation over half-period hereafter called modulation impulse. This impulse is a quantity that accounts simultaneously for the modulation's amplitude, period and waveform.

The dynamics of our model system of ultracold atoms interacting with a phase modulated light field produced using an oscillating mirror is well described by the periodic Hamiltonian

$$\widetilde{H} = \widetilde{p}^2/(2M) - V_0 \cos\left[2k\widetilde{x} - \lambda F\left(t\right)\right], \qquad (1)$$

where M is the atomic mass, $\widetilde{x}$ the position, $\widetilde{p}$ the momentum, V_0 the potential height, k the wave number, λ the dimensionless modulation depth, and $F\left(t\right)$ the AC modulation given by

$$F(t) = F(t;m,T) \equiv N(m)\,\mathrm{sn}\left[4Kt/T\right]\mathrm{dn}\left[4Kt/T\right], \qquad (2)$$

where $\mathrm{sn}\left(\cdot\right) \equiv \mathrm{sn}\left(\cdot;m\right)$ and $\mathrm{dn}\left(\cdot\right) \equiv \mathrm{dn}\left(\cdot;m\right)$ are Jacobian elliptic functions of parameter m, and $K \equiv K(m)$ is the complete elliptic integral of the first kind. $N(m)$ is the normalization factor given by

$$N(m) \equiv \left\{a + \frac{b}{1+\exp[(m-c)/d\,]}\right\}^{-1}, \qquad (3)$$

where the values of the parameters are set equal to $a \equiv 0.43932$, $b \equiv 0.69796$, $c \equiv 0.3727$, and $d \equiv 0.26883$, in order to have the same modulation amplitude (equal to unity) and period, T, independently of the waveform, i.e. $\forall m \in [0,1]$. Notice that F in (1) introduces a convenient (since allowing obtaining many analytical results) flexible periodic pulse which varies its form depending on the value of m. For $m = 0$, one recovers the well known harmonic excitation case since then $F\left(t; m=0, T\right) = \sin\left(2\pi t/T\right)$. On the other hand, for $m \neq 0$ the waveform has different shapes. For example, for $m = 0.72$ a nearly square–wave pulse is obtained, whereas for the limiting value $m = 1$ the modulation vanishes.

As will be shown, the modulation impulse associated with $F(t)$, defined as

$$I \equiv I(m,T) = \int_0^{T/2} F(t;m,T)dt = \frac{TN(m)}{2K(m)}, \qquad (4)$$

is a relevant quantity to characterize the effect of the modulation's waveform. As expected, it is a function of m, which has a single maximum at $m = m^I_{\max} \simeq 0.717$. Also, it tends to zero very quickly as $m \to 1$.

Our results show for AC-driven space-periodic Hamiltonians that the impulse transmitted by the timeperiodic modulation is an essential quantity to understand the phenomenon of DL, which can be optimally controlled by changing the form of the AC, i.e. the parameter m in Eq.(2). Remarkably, our results shown that the impulse principle reliably control DL irrespective of the quanticity degree of the system. While this result holds for the wide class of AC modulations having equidistant consecutive zeros, one expects it to be also valid for even more general class of time-periodic modulations, at least in the adiabatic regime. This principle, which can be straightforwardly applied to other phenomena, such as field-induced barrier transparency or quasi-energy band collapse, paves the way for optimum coherent control of diverse quantum systems. Additionally, it has been shown that the classical invariant structures of phase space have a deep impact on the quantum dynamical behavior of the system, as the modulation impulse is varied.

[1] F. Revuelta, R. Chacón, and F. Borondo, EPL **110**, 40007 (2015).