Catastrophes in optics: Analysing the dynamics and stability of caustic beams

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Catastrophe science [2] is a branch of bifurcation theory in the study of nonlinear dynamical systems; it is also a special case of more general singularity physics. Bifurcation theory studies and classifies phenomena characterized by sudden shifts in behaviour arising from small changes in circumstances, analysing how the qualitative nature of solutions depends on external control parameters. In singular optics, modelling the distribution of light according to these nonlinear potentials $P_n(\mathbf{a}, s)$ via the so-called catastrophe diffraction integral $C_n(\mathbf{a})$ leads to the formation of caustics as geometrically stable structures [1]. Emerging as natural phenomena, they are associated with the arcs close to rainbows, or may occur as ramified high-intensity networks on the floor of shallow waters. Similar to their formation behind refractive index lenses with imperfections, the occurrence of corresponding structures has been observed for numerous kinds of lenses with importance in optics, astrophysics and surface analytics.

In recent years, known catastrophic light structures that were already observed in natural phenomena in the late seventies and eighties [1] experience a renaissance since they are embedded in artificially designed caustic beams that depend on a single state parameter s and external control parameters $\mathbf{a}$:

$$C_n(\mathbf{a}) = \int\limits_R e^{\mathrm{i}P_n(\mathbf{a},s)}\mathrm{d}s, \quad P_n(\mathbf{a}, s) = s^n + \sum_{j=1}^{n-2} a_j s^j. \quad (1)$$

The most prominent representative of a catastrophe that manifests as wave package is the fundamental fold catastrophe that has been realized as paraxial Airy beam $\mathrm{Ai}(x) = C_3(x)$ in 2007 [3]. It shows a transverse-invariant and accelerated propagation on a parabolic trajectory. Benefiting from these striking properties, many linear and nonlinear applications arise in different branches of photonics like particle manipulation or signal transferring [4, 5]. In 2012, the second fundamental cusp catastrophe was transferred to optics as paraxial Pearcey beam $\mathrm{Pe}(x, y) = C_4(x, y)$, and attracts attention due to its auto-focusing and form-invariant propagation [6].

Further, designing complex caustic light supports the field of nonlinear photonics which experiences explosive growth during the last decade and became a vital branch in modern photonics. Combining both topics opens up brand new aspects of research and enables the investigation of light guiding and localized Airy and caustic photonic structures [5].

In our contribution, we report on our recent results about embedding higher-order catastrophes, like the swallowtail and hyperbolic umbilic catastrophe [1, 2], in paraxial light. Our approach connects two control parameters with the transverse spatial coordinates, thus cross-sections through the control parameter space are mapped to the caustic light structure. We analytically derive an equation capable to calculate in general the propagation of paraxial caustic beams that depend on a single state parameter, and show that the propagation of a caustic beam of order n can be expressed in terms of stationary higher-order caustic beams of order $2(n-2)$. To demonstrate this, we thoroughly study the dynamics of different swallowtail caustics analytically and experimentally, and link them with stationary higher-order butterfly catastrophes. Simultaneously, we analyze the stability of the swallowtail caustic. It decays to a cusp during propagation because its initial field was created as cross-section through the stable complete swallowtail catastrophe.

Subsequently, utilizing higher-order cusp and swallowtail catastrophes in paraxial light to inscribe photonic caustic structures in photosensitive media, we demonstrate waveguiding with a rich diversity of light guiding paths. Taking advantage of the strong auto-focusing of caustic beams in photorefractive nonlinear crystals, we realized the formation of Pearcey solitary waves.

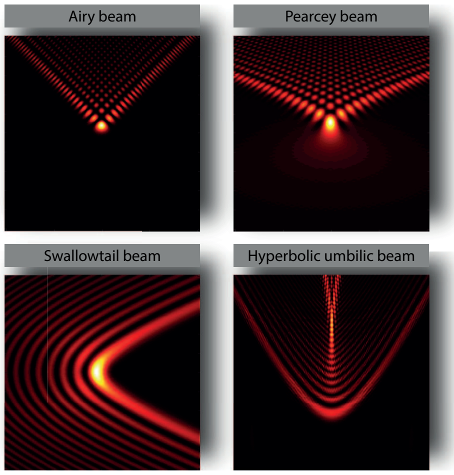

Figure 1: Caustic beams with embedded elementary catastrophes: fold, cusp, swallowtail, and hyperbolic umbilic.

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