Bright and dark localized structures in the Lugiato-Lefever equation

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In this work we present a detailed analysis of the bifurcation structure of localized structures (LSs) and their different dynamical regimes in the Lugiato-Lefever equation in the presence of anomalous and normal chromatic dispersion. Such analysis is expected to also provide new insights into the formation and stability of frequency combs (FCs). A FC consists in a set of equidistant spectral lines that can be used to measure light frequencies and time intervals more easily and precisely than ever before [1]. Due to the shown duality between LSs and FCs in microcavities, we can gain information about the behavior of FCs by analyzing the dynamics of LSs. In the anomalous dispersion case bright LSs are organized in what is known as a homoclinic snaking bifurcation structure [2, 3]. In contrast, in the normal dispersion regime dark LSs exist and they are organized differently, in a structure known as collapsed snaking [4, 5]. Despite the differences in bifurcation scenarios, both types of LSs present similar temporal instabilities. The similarities and differences between both scenarios correspond to two different unfoldings of the same set of codimension-two bifurcations that organize all the dynamics of the system. This work is expected to be useful to experimentalists working in the domain of FC generation as it provides a detailed map of the different dynamical regimes of LSs and FCs in microresonators. We will show how higher order effects modify the previous scenarios [6, 7].

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