NestModularity measure for the join analysis of nested and modular networks

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In the past years the concept of nestedness has clearly overflowed the classical ecological framework: beyond mutualistic networks, we have now evidences that nested patterns appear in diverse settings, from anthropology and sociology to economy and urban science. Parallel to the discovery of new instances of nested organizations, scholars have debated around the co-existence of two apparently incompatible mesoscale structures: nestedness and modularity. In this regard, the discussion is far from a solution mainly for two reasons. First, nestedness and modularity appear to be the result of two contradictory dynamics, a competitive and a cooperative one [1]. Second, existing methods to evaluate the presence of nestedness and modularity are flawed when it comes to the evaluation of concurrently nested and modular structures. In this work, we define the concept of NestModularity as a structural network measure that assesses to what extent the network is composed of modules where the relationship of elements within the modules exhibit a nested structure. To do so, we have developed a fitness function NQ that, given a membership variable α for rows and columns, evaluates the quality of a partition of the network into nested blocks. Function NQ incorporates the definition of nestedness (NODF [2] in particular) into a modularity-like formulation that considers the difference between the observed, O_{ij}, and the expected overlap, $\langle O^r_{ij} \rangle$. See Eq. 1.

$$\mathrm{NQ} = \frac{2}{M+N} \left(\sum_{ij}^{N} \frac{O^r_{ij} - \langle O^r_{ij} \rangle}{k^r_j \left(N_{\alpha^r_i} - 1 \right)} \delta\left(\alpha^r_i, \alpha^r_j\right) H(k^r_i - k^r_j) + \right.$$
$$\left. + \sum_{ij}^{M} \frac{O^c_{ij} - \langle O^c_{ij} \rangle}{k^c_j \left(M_{\alpha^c_i} - 1 \right)} \delta\left(\alpha^c_i, \alpha^c_j\right) H(k^c_i - k^c_j) \right) \quad (1)$$

where k is the degree, δ is the Kronecker delta, H is the Heaviside function and sup-indexes r and c indicate row and column. The size of the system $M + N$ (number of rows and columns) appears for proper normalisation.

Our analysis on real networks have proven both the existence of such mesoscale structures and that our methodology is capable of detecting them. Results shown in Fig. 1 illustrates our methodology and shows that the current approach of analyzing modularity and nestedness separately can, among other intrinsic problems, easily underestimate nestedness and overestimate modularity.

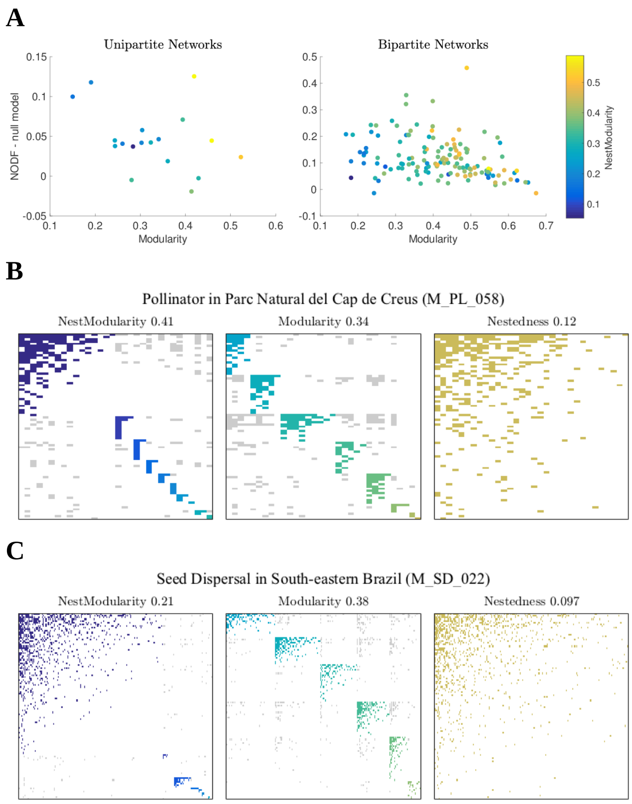

Figure 1: (A) Scatter plot confronting modularity and nestedness with the NestModularity measure (color coded). Each point represents either an ecological, urban or social network (left panel: unipartite networks; right panel: bipartite networks). (B) Example where nestedness is underestimated because of the existence of modules in the network (source: http://www.web-of-life.es). (C) Example where classical modularity overestimates the number of compartments in an ecological network (source: http://www.web-of-life.es).

[1] Thébault, Elisa, and Colin Fontaine. "Stability of ecological communities and the architecture of mutualistic and trophic networks." Science 329.5993 (2010): 853-856.

[2] Almeida-Neto, Mário, et al. "A consistent metric for nestedness analysis in ecological systems: reconciling concept and measurement." Oikos 117.8 (2008): 1227-1239.