Geometric renormalization of real complex networks

Guillermo García-Pérez[1,2], Marián Boguñá[1,2] and M. Ángeles Serrano[1,2,3]

[1]Departament de Física de la Matèria Condensada, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain
[2]Universitat de Barcelona Institute of Complex Systems (UBICS), Universitat de Barcelona, Barcelona, Spain
[3]ICREA, Pg. Lluís Companys 23, E-08010 Barcelona, Spain

Renormalization has proven to be a very powerful tool for a systematic investigation of systems–from condensed matter to quantum field theory–as viewed at different scales. In the field of complex networks, some topological renormalization schemes based on shortest paths between nodes have been proposed in the past [1]. However, the collection of shortest-path lengths, albeit a well-defined metric space, is a poor source of length-based scaling factors in networks due to their homogeneity and the small-world property.

We present empirical evidence that real-world complex networks are invariant under geometric length scale transformations when embedded in an underlying hidden metric space. We prove analytically [2] that the embedding model [3, 4] is also renormalizable in the same framework and take it as new evidence supporting our conjecture that hidden metric spaces underlie real networks. The congruency between real networks and the geometric model enables to define a multiscale unfolding of complex networks that allows the study of their self-similarity properties.

As applications, this geometric renormalization scheme yields a natural way of building smaller-scale replicas of real networks while simultaneously preserving their statistical properties, which can be extremely useful in the study of dynamical processes on large networks. The hidden metric space renormalization group can also be applied to design a new greedy routing protocol in hyperbolic space which exploits the multiscale unfolding of complex networks increasing the success rate of single-scale versions.

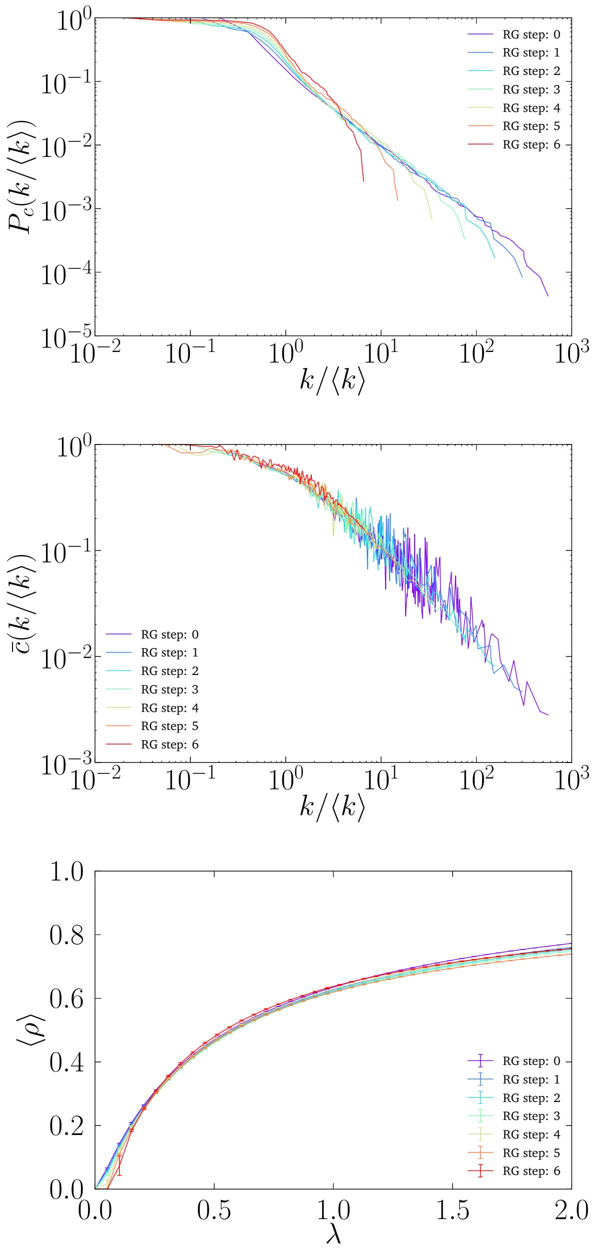

Figure 1: **Geometric renormalization of the Internet AS network.** The plots compare the networks obtained from the geometric renormalization transformations. At every renormalization step, the number of nodes is reduced by a factor 2. **Top:** Rescaled complementary cumulative degree distributions. **Middle:** Rescaled clustering coefficient spectra. **Bottom:** Average prevalence as a function of the infection rate in the SIS dynamics on the original network and its smaller-scale replicas.

[1] C. Song, S. Havlin, and H. A. Makse, Nature **433**, 392 (2005).

[2] In preparation.

[3] M. Á. Serrano, D. Krioukov, and M. Boguñá, Phys. Rev. Lett. **100**, 078701 (2008).

[4] F. Papadopoulos, M. Kitsak, M. Á. Serrano, M. Boguñá, and D. Krioukov, Nature **489**, 537 (2012).