Quantum Brownian Motion Revisited

Aniello Lampo[1], Soon Hoe Lim[2], Jan Wehr[2], Pietro Massignan[1], and Maciej Lewenstein[1,3]
[1]ICFO - Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain
[2]Department of Mathematics, University of Arizona, Tucson, Arizona 85721-0089, USA
[3]ICREA, Lluis Companys 23, E-08010 Barcelona, Spain

Brownian motion in an inhomogeneous medium has been recently studied in the context of classical Brownian motion (CBM) and other classical diffusive systems. Precisely, explicit formulas were derived for noise-induced drifts in the small-mass (Smoluchowski-Kramers [1, 2, 3] and other limits).

In a series of papers [4, 5, 6] we analyzed the microscopic model of quantum Brownian motion, describing a Brownian particle interacting with a bosonic thermal bath, through a coupling which is linear in the creation and annihilation operators of the bath, but may be a nonlinear function of the position of the particle. Physically, this corresponds to a configuration in which damping and diffusion are inhomogeneous, namely they vary in space.

In [4] we derive systematically the quantum master equation for the Brownian particle in the Born-Markov regime, discussing the appearance of additional terms for various polynomial forms of the coupling. We pay particular attention to the cases of linear and quadratic coupling, studying, by means of a Wigner function techniques, the stationary state of a Brownian particle in a harmonic trapping potential. We predict quite generally Gaussian stationary states, and we compute the aspect ratio and the spread of the distributions. In particular, we find that these solutions may be squeezed with respect to the position of the Brownian particle, corresponding to its high spatial localization. We further describe the dynamical stability of the system, by applying a Gaussian approximation to the time-dependent Wigner function, and we calculate the decoherence rates of coherent quantum superpositions in position space. Part of the work is devoted to analyze various restrictions to the validity of our theory posed by Born and Markov approximations, leading to a breakdown of positivity of the density operator at very low-temperature and strong coupling, associated to a violation of the Heisenberg principle.

To overcame the violation of the positivity of the solution of a Born-Markov master equation in [4], we recall a Lindblad one, constructed in order to be free of such a problem. The results have been presented in [5]. We study the dynamics of the model, including the detailed properties of its stationary solution, for both linear and non-linear coupling of the Brownian particle to the bath, focusing in particular on the correlations and the squeezing of the probability distribution induced by the environment.

An immediate application of the theory of quantum Brownian motion in an inhomogeneous medium concerns the problem of an impurity in an ultracold gas, intensively studied in the context of Bose polaron. This is the purpose of the work in [6]. Here we treat the dynamics of the system by means of Heisenberg equations, in order to go beyond the approximations in [4, 5]. We solve these equations, looking to the position variance of the impurity, that may be measured in experiments for the present system. We distinguish the situation where the impurity is trapped in a harmonic potential and the case where there is no trap. In the first case, the impurity approaches an equilibrium, and its final stationary state genuine position squeezing, corresponding to high spatial localization (Fig. 1). In the situation in which there is no trap, the impurity does not approach an equilibrium. Here the position variance depends on time, and it manifests a super-diffusive behavior, precisely $< x^2 >\sim t^2$. Such a feature represents a consequence of the presence of memory effects.

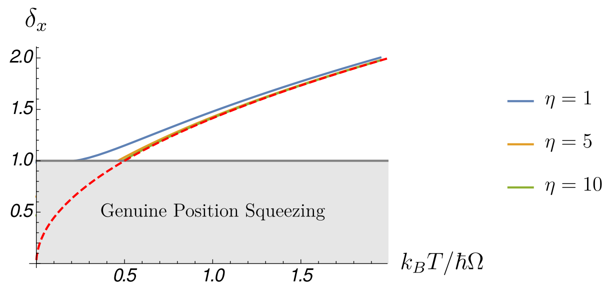

Figure 1: Temperature dependence of the position variance of an impurity of K trapped in a harmonic potential with $\Omega = 2\pi \cdot 250$Hz, in a gas made up by Rb with a density of $n_0 = 7(\mu m_{\rm I})^{-1}$ and a coupling strength among the atoms $g_{\rm B} = 2.36 \cdot 10^{-37}$Jm. The lines refer to different value of $\eta = g_{\rm IB}/g_{\rm BB}$, where $g_{\rm IB}$ is the coupling constant related to the interaction between the impurity and the atoms of the gas.

[1] G. Volpe, L. Helden, T. Brettschneider, J. Wehr, and C. Bechinger, Influence of Noise on Force Measurements, Phys. Rev. Lett. **104**, 170602 (2010).

[2] G. Pesce, A. McDaniel, S. Hottovy, J. Wehr, and G. Volpe, Stratonovich-to-Ito transition in noisy systems with multiplicative feedback, Nature Comm. **4**, 2733 (2013).

[3] Scott Hottovy, Austin McDaniel, Giovanni Volpe, and Jan Wehr, The Smoluchowski-Kramers Limit of Stochastic Differential Equations with Arbitrary State-Dependent Friction, Commun. Math. Phys. **336**, 1259 (2015)

[4] Pietro Massignan, Aniello Lampo, Jan Wehr, and Maciej Lewenstein, Quantum Brownian motion with inhomogeneous damping and diffusion, Phys. Rev. A **91**, 033627 (2015)

[5] Aniello Lampo, Soon Hoe Lim, Jan Wehr, Pietro Massignan, and Maciej Lewenstein, Lindblad model of quantum Brownian motion, Phys. Rev. A **94**, 042123 (2016).

[6] Aniello Lampo, Soon Hoe Lim, Jan Wehr, and Maciej Lewenstein, Bose polaron as an instance of quantum Brownian motion, in preparation (2017).