Hierarchical Partitions of Social Networks Between Rivaling Leaders

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A model algorithm is proposed to study subsequent hierarchical partitions of complex networks describing social structures. The partitions are supposed to appear as actions of rivaling leaders [1] corresponding to nodes with large degrees. The condition of a partition is that the distance between two leaders is at least three links. This ensures that the layer of nearest neighbours of each leader remains attached to him. The process of cutting links starts from a selection of the shortest path between the leaders. If there is more than one path, we concentrate on one of them. If the length of the path is exactly three, there is only one link in the middle to be cut. If the shortest path consists of more than three links, the cutting can be performed in two ways; either we select the link with the lowest number (variant A) or the link with the highest number (variant B). As a rule, numerically calculated size distribution of fragments of scale-free Albert-Barabasi networks reveals one large fragment which contains the original leader (hub of the network), and a number of small fragments with opponents (see Fig. 1) that are described by two Weibull distributions [2].

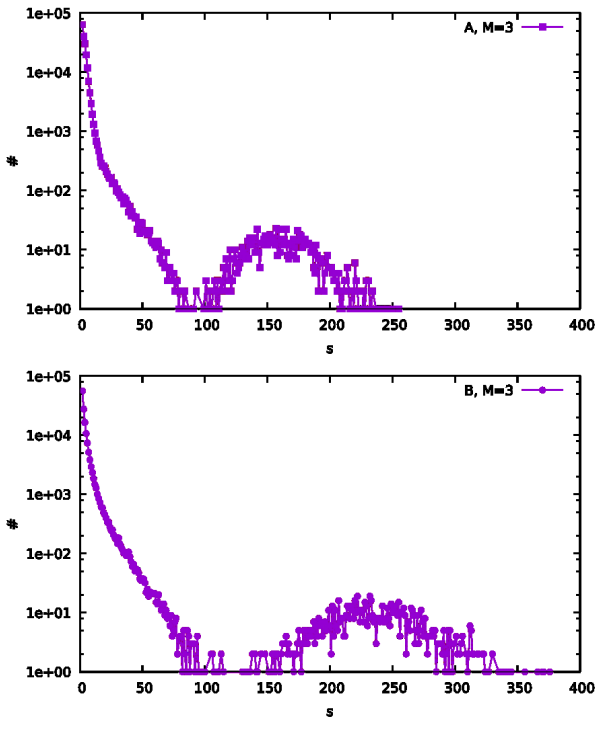

Figure 1: The histogram of the fragment size s distribution for variants A and B of hierarchical partition (top/bottom plots) of Albert-Barabasi model with a parameter $M = 3$.

Numerical simulations and mean-field theory reveal that size of the larger fragment scales as the square root of the initial network size. The algorithm is applied to the data on political blogs in U.S. (L. Adamic and N. Glance, Proc. WWW-2005). The obtained fragments are clearly polarized; either they belong to Democrats, or to the GOP. (see Fig. 2)

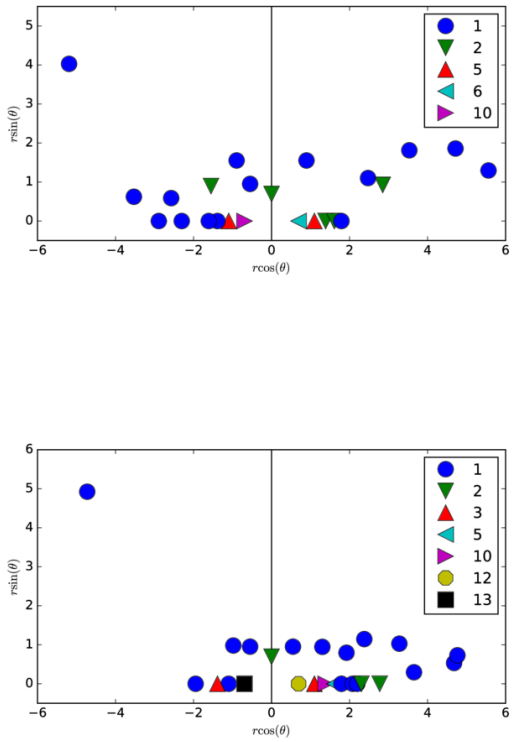

Figure 2: The fragments obtained by the partition of the network of political blogs [3]. The position of a fragment is related to its size and its content: $r = \log(N_R + N_D)$, and $\Theta = \pi N_D/(N_R + N_D)$, where N_D (N_R) is the number of blogs tagged as democratic (republican). The same data on the blogs [3] are used twice for various versions of our partition algorithm.

[1] K. Kacperski and J.A. Holyst, Physica A: Statistical Mechanics and its Applications. 2000; **287**(34):631–643.

[2] Malgorzata J. Krawczyk, Krzysztof Kulakowski and Janusz A. Holyst, arXiv:1611.05604.

[3] L.A. Adamic and N. Glance, *The Political Blogosphere and the 2004 U.S. Election: Divided They Blog.* In: Proceedings of the 3rd International Workshop on Link Discovery. LinkKDD '05. New York, NY, USA: ACM; 2005. p. 36–43.